Foreign Exchange Rate Arbitrage using the Matrix Method

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บทคัดย่อ

บทความนี้ ว่าด้วยการค้นหาโอกาสค้ากำไรในตลาดเงินตราต่างประเทศ วิธีดั้งเดิมที่ใช้ในการบ่งชี้โอกาสค้ากำไรนั้น อาจมีประสิทธิภาพต่ำเมื่อจำนวนสกุลเงินที่เราพิจารณาเพิ่มมากขึ้น ดังนั้น จึงเกิดความจำเป็นที่จะต้องหาวิธีที่มีประสิทธิภาพที่ใช้ในการค้นหาเส้นทางค้ากำไรที่ประกอบไปด้วยสกุลเงินหลายสกุล

หม่า (2004) ได้พัฒนาวิธีเมตริกซ์ที่มีประสิทธิภาพในการค้นหาเส้นทางการค้ากำไรในตลาดที่ประกอบไปด้วยสกุลเงิน N สกุล หม่า ยังได้ค้นพบเงื่อนไขเพียงพอสำหรับการมีอยู่ของโอกาสค้ากำไรด้วย โดยที่งานของหม่ามีพื้นฐานอยู่บนสมมติฐานที่ว่านักค้ากำไรสามารถซื้อและขายเงินตราแต่ละสกุลที่ราคาเดียวกัน อย่างไรก็ตาม ในความเป็นจริง ส่วนต่างราคาเสนอซื้อขายถือเป็นองค์ประกอบสำคัญของค่าใช้จ่ายในการซื้อขายเงินตราต่างประเทศ ดังนั้น เราควรคำนึงถึงส่วนต่างราคาเสนอซื้อขายในขั้นตอนการค้นหาเส้นทางค้ากำไรด้วย

ในบทความนี้ ผู้เขียนปรับเปลี่ยนวิธีเมตริกซ์ของหม่า เพื่อที่จะรองรับกรณีที่ส่วนต่างราคาเสนอซื้อขายไม่เป็นศูนย์ หลังจากนั้น บทความนี้ยังแสดงขั้นตอนการประยุกต์วิธีนี้ โดยมีข้อมูลอัตราแลกเปลี่ยน 65 วัน เป็นกรณีตัวอย่าง โดยมีข้อสังเกตว่า เงื่อนไขเพียงพอที่ถูกค้นพบในงานของหม่านั้น ยังคงใช้ได้อยู่แม้กระทั่งในกรณีที่ส่วนต่างราคาเสนอซื้อขายไม่เป็นศูนย์

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Abstract

This article focuses on finding arbitrage opportunities in the international foreign exchange market. The traditional methods of distinguishing arbitrage opportunity have low efficiency, and difficulties arise when we face multiple currencies. This poses a need for an efficient method of searching for a multiple-currency arbitrage path.

Ma (2004) has developed a matrix method that efficiently searches for an arbitrage path in an N-currency market. Ma also derives a simple sufficient condition that will guarantee the presence of arbitrage opportunities. Ma's work is based on the assumption that one can buy or sell each currency at zero bid-ask spread. However, the bid-ask spread represents an important component of the transaction costs, and therefore should be incorporated into the search for arbitrage path.

In this article, the author modifies Ma's matrix method to accommodate the case where the bid-ask spreads are nonzero. We then illustrate the use of the modified method, using a 65-day series of exchange rate quotes. We also observe that the sufficient condition derived in Ma's work still holds even for the case of nonzero bid-ask spread.

Key words : Matrix method, Arbitrage, bid-ask spread

1 Introduction

1.1 Background

Arbitrage is the most important concept in finance and is a fundamental mechanism for achieving efficiency in the financial markets (Ross 1976). An arbitrage opportunity occurs when a price discrepancy exists between two or more highly related assets. It is commonly assumed that state variables of financial instruments will disallow the existence of investment strategies with riskless profit. Active investors who have rapid identification, fast transactions, and low transaction costs will exploit any arbitrage opportunity in a financial market by buying the underpriced asset and selling the overpriced asset without any risk. Therefore, mispricing is rapidly corrected in highly competitive markets (Frenkel and Levich 1975,1977).

Foreign exchange market is an important part of the financial market. In the global foreign market, there will be the arbitrage opportunity due to the geographically separated market, difference of operation time and information. Exchange rate arbitrage is the practice of taking advantage of inconsistent exchange rates in different markets by selling in one market and simultaneously buying in another. Examination of data from ten markets over a 12-day period by Mavrides (1992) revealed that a significant arbitrage opportunity exists. Some opportunities were observed to be persistent for a long time. There are two types of arbitrage to the forex markets: Exchange rate arbitrage and Interest rate arbitrage. Exchange rate arbitrage involved two-point and three-point arbitrage. Two-point arbitrage concerns two currencies in two different markets. Three-point arbitrage is commonly called triangular arbitrage, that is, exchange rates among different currencies may be mutually inconsistent. Arbitrageurs will then attempt to profit from these inconsistencies and in the process will eliminate discrepancies and establish mutually consistent cross-exchange rates. One can choose any currency to start.A successful arbitrage depends on whether the currency one starts with is the same as the end currency.

In most financial markets there are always two prices for a particular financial instrument at any one time which are known as the bid and the ask price. The bid price is the price at which the market maker (the actual entity that is on the other side of the trade) will buy and therefore the rate at which you the client can sell. The ask price is the price at which a market maker will sell and therefore the rate at which you, the client, can buy. The difference between the rate at which you can sell (the bid) and the rate at which you can buy (the ask) is referred to as “the spread”. for example, the current bid price for the EUR/USD currency pair is 1.5760 and the current ask price is 1.5763. This means that currently you can sell the EUR/USD at 1.5760 and buy at 1.5763. The difference between those prices is the spread. The bid-ask spreads exist in the real market. They represent an important component of the transaction costs.

1.2 Objective

This article tries to find the arbitrage opportunity with an efficient way in the international foreign exchange market. Because of the difficulty of the position, operating time and information, one needs an efficient algorithm to search for a multiple-currency arbitrage path.

Ma (2004) has developed the matrix method that efficiently searches for an arbitrage path in an N-currency market. Ma also derives a simple sufficient condition that will guarantee the presence of arbitrage opportunities. When$\lambda max=n$ there exists the arbitrage opportunity. Because the assumption of Ma's work bases on the zero bid-ask spread and the bid-ask spread is an important component of the transaction cost, we try to test whether$\lambda max\neq n$ still hold for the case with bid-ask spread, therefore, we need to modify method of Ma to get a new conclusion.

In this article, the author modifies the matrix method to accommodate the case where the bid-ask spread are nonzero. We gather three month data to illustrate the use of the modified method and find that the sufficient condition derived from Ma's work still holds for the case of the bid-ask spread.

2 Literature Review

Although the transaction of foreign exchange market has very developed with electronic system, the theory of foreign exchange arbitrage still heritage the triangular arbitrage theory in 1970s and takes no furthur step for many years. Recently, Bollard and Connor(1996) adapt Kalman filter to filter tick data, copy with the erratic arrival of observation and produce estimates of all the arbitrage prices on every time step. The filter produces estimates of the arbitrage price for all exchange rates on every second, increasing both the speed and efficiency of arbitrage identification. Mao-cheng Cai and Xiaotie Deng (2003) study the computational complexity of arbitrage in frictional foreign exchange markets with bid-ask spread, bound and integrality constraints.Constrast to the complexity calculation of papers above, Ming Ma (2004) applies matrix to his analysis, in his paper, the arbitrage-free benchmark matrix B is constructed from real forex matrix A, matrix C reveals measures for deviation of each currency from its benchmark value which indicates the possibly optimal arbitrage path.

3 Modifying Ma's Matrix Method to Accommodate the Presence of Bid-Ask Spread

3.1 Data Collection

This article focuses on the arbitrage opportunity with six currency (USD, JPY, GBP, SGD,HKD,EUR) in six markets (New York, Frankford, Singapore, Tokyo, London, Hong Kong). For the three month study period from Sep 1, 2008 to Nov 28, 2008, the daily exchange rate data from Datastream. Note that the data for each exchange rate is not synchronized. For example, the daily HKD/USD rate and the USD/GBP rate are not collected at the same time of day. However, we

shall use this data set to illustrate the procedure of determining an arbitrage path.

3.2 Analytical Framework

Quantitative analysis is used to analyze the association between the arbitrage path, maximum bid-ask spread, profit and currency which be underpriced or overpriced. This analysis is based on Ma's matrix method, but some adjustments is made in order to accommodate the presence of the bid-ask spread.

This research performs three different matrix to analyze how the arbitrage path can be found with or without the bid-ask spread. Three month data is used. Both analysis are performed to show the arbitrage path, profit and find the currency which be underpriced or overpriced, which can provide the optimal path and maximum profit for the arbitrage. We now illustrate the method for the case with bid-ask spread, the raw data is as follows.

Next, we calculate the λmax and eigen values and eigenvectors associated with λmax of matrix A with MATLAB, So we get λmax =6.0221 and G=[g1, g2....gn]=[0.3802, 0.5548, 0.6881, 0.0035, 0.049, 0.2678], because the exchange rate of any two currency is determined by their gold contend, therefore, there is no arbitrage opportunity and λ of matrix B is equal to n, where matrix B is the arbitrage free benchmark. We can prove (see Ma 2004) that the sufficient condition of no arbitrage is λmax=n and the eigen value G is the gold contend. Here, we get the conclusion that there is arbitrage opportunity if λmax not equal to n.

Table 3-1 Raw data with bid price and ask price

Currency pair	Price	Transaction date (65 days)						
		2008-9-1	2008-9-2	2008-9-3	2008-9-4	.....	2008-11-27	2008-11-28
USD / EUR	bid	1.4585	1.4514	1.4491	1.4241	.....	1.2894	1.269
	ask	1.459	1.4519	1.4496	1.4246	.....	1.2899	1.2695
USD / GBP	bid	1.7993	1.7829	1.775	1.76	.....	1.5393	1.5381
	ask	1.7998	1.7834	1.7755	1.7605	.....	1.5398	1.5386
HKD / USD	bid	7.8054	7.8071	7.8075	7.8072	.....	7.7516	7.7502
	ask	7.8064	7.8081	7.8085	7.8082	.....	7.7526	7.7512
JPY / USD	bid	108.13	108.58	108.21	106.41	.....	95.31	95.53
	ask	108.18	108.64	108.25	106.46	.....	95.36	95.58
SGD / USD	bid	1.4237	1.4325	1.4351	1.4396	.....	1.5095	1.5133
	ask	1.4247	1.4335	1.4361	1.4406	.....	1.5105	1.5143
HKD / EUR	bid	11.3859	11.3356	11.2887	11.2284	.....	9.9793	9.8323
	aks	11.3894	11.3387	11.2918	11.2313	.....	9.9823	9.836
JPY / EUR	bid	157.7255	158.0066	156.7573	155.4083	.....	123.0111	120.8183
	aks	157.8017	158.0974	156.8332	155.4839	.....	123.0912	120.8945
SGD / EUR	bid	2.0775	2.0758	2.0765	2.063	.....	1.9423	1.9133
	ask	2.0789	2.0772	2.0778	2.0644	.....	1.9435	1.9148
GBP / EUR	bid	0.8109	0.8128	0.8137	0.8114	.....	0.8353	0.8267
	ask	0.8113	0.8132	0.814	0.8118	.....	0.8357	0.8272
JPY / GBP	bid	192.35	190.7	189.97	188.39	.....	142.24	143.12
	ask	200.35	198.7	197.97	196.39	.....	150.24	151.12
JPY / SGD	bid	75.66	75.23	75.14	74.84	.....	62.2	62.63
	ask	77.32	76.89	76.8	76.5	.....	63.86	64.29
JPY / HKD	bid	13.49	13.47	13.51	13.44	.....	11.87	11.89
	ask	14.35	14.33	14.37	14.3	.....	12.73	12.75
HKD / SGD	bid	5.477	5.477	5.421	5.4575	.....	5.128	5.141
	ask	5.4845	5.4845	5.4285	5.4655	.....	5.134	5.147
HKD / GBP	bid	14.095	14.095	13.825	13.895	.....	11.93	11.955
	ask	14.11	14.11	13.84	13.91	.....	11.945	11.97
SGD/GBP		2.5657	2.5549	2.5501	2.5419	.....	2.3255	2.3261
		2.5657	2.5549	2.5501	2.5419	.....	2.3255	2.3261

From the raw data, We pick Sep 1, 2008 as the example to construct matrix A, B, and C.

First, we build the matrix A with bid price and ask price,. For example, 1.459 is the ask price of USD/EUR, 0.685636 equal to 1/1.4585, where the 1.4584 is the bid price of USD/EUR.

A	2008-9-1	USD	EUR	GBP	JPY	HKD	SGD
	USD	1	0.685636	0.555772	108.18	7.8064	1.4247
	EUR	1.459	1	0.8113	157.8017	11.3894	2.0789
	GBP	1.7998	1.233198	1	200.35	14.11	2.5657
	JPY	0.009248	0.00634	0.005199	1	0.074129	0.013217
	HKD	0.128116	0.087828	0.070947	14.35	1	0.182582
	SGD	0.702395	0.481348	0.389757	77.32	5.4845	1

$$B = \begin{bmatrix} \frac{g_1}{g_1} & \frac{g_1}{g_2} & \cdots & \frac{g_1}{g_n} \\ \frac{g_2}{g_1} & \frac{g_2}{g_2} & \cdots & \frac{g_2}{g_n} \\ \vdots & \vdots & \vdots & \vdots \\ \frac{g_n}{g_1} & \frac{g_n}{g_2} & \cdots & \frac{g_n}{g_n} \end{bmatrix}$$, we get matrix B ,

B	2008-9-1	USD	EUR	GBP	JPY	HKD	SGD
	USD	1	0.685292	0.552536	108.6286	7.759184	1.419716
	EUR	1.459232	1	0.806278	158.5143	11.32245	2.071695
	GBP	1.809837	1.240267	1	196.6	14.04286	2.569455
	JPY	0.009206	0.006309	0.005086	1	0.071429	0.013069
	HKD	0.12888	0.08832	0.071211	14	1	0.182972
	SGD	0.704366	0.482696	0.389188	76.51429	5.465306	1

Last, with the formula C=A÷B, (where÷means element-by-element division), we get matrix C. for example, 0.99984=1.459/1.459232.

C	2008-9-1	USD	EUR	GBP	JPY	HKD	SGD
	USD	1	1.0005	1.005856	0.995871	1.006085	1.00351
	EUR	0.99984	1	1.006228	0.995505	1.005913	1.003478
	GBP	0.99445	0.9943	1	1.019074	1.004781	0.998539
	JPY	1.00461	1.005	1.022095	1	1.037806	1.011291
	HKD	0.99408	0.99443	0.996301	1.025	1	0.997865
	SGD	0.9972	0.99721	1.001463	1.01053	1.003512	1

Table 3-2 Arbitrage with bid-ask spread

	λmax	Path	Profit(%)	Underpriced	Overpriced
2008-9-1	6.0221	EUR-GBP-HKD-EUR	0.353	GBP/ EUR HKD/ GBP	EUR/HKD
2008-9-2	6.0222	USD-GBP-HKD-USD	1.221	GBP/USD HKD/GBP	USD/HKD
2008-9-3	6.0221	HKD-GBP-EUR-HKD	0.381	GBP/HKD HKD/ EUR	EUR/GBP
2008-9-4	6.0224	USD-EUR-JPY-USD	2.469	EUR/USD JPY/ EUR USD/JPY	
2008-9-5	6.0227	USD-JPY-EUR-USD	1.141	JPY/USD USD/ EUR	EUR/JPY
2008-9-8	6.0223	USD-GBP-HKD-USD	1.055	GBP/USD HKD/GBP	USD/HKD
2008-9-9	6.0225	USD-EUR-JPY-USD	0.706	EUR/USD JPY/ EUR	USD/JPY
2008-9-10	6.0226	USD-GBP-HKD-USD	0.798	GBP/USD HKD/GBP	USD/HKD
2008-9-11	6.0227	USD-JPY-EUR-USD	1.246	JPY/USD USD/ EUR	EUR/JPY
2008-9-12	6.0227	USD-SGD-GBP-USD	1.867	SGD/USD GBP/SGD USD/GBP	
2008-9-15	6.0227	USD-HKD-GBP-USD	1.569	HKD/USD GBP/HKD	USD/GBP
2008-9-16	6.0231	EUR-GBP-HKD-EUR	1.128	GBP/ EUR HKD/GBP	EUR/HKD
2008-9-17	6.0228	USD-HKD-GBP-USD	0.845	HKD/USD	GBP/HKD, USD/GBP
2008-9-18	6.023	USD-EUR-SGD-USD	0.309	EUR/USD SGD/ EUR	USD/SGD
2008-9-19	6.0227	EUR-HKD-GBP-EUR	1.484	HKD/EUR EUR/GBP GBP/HKD	
2008-9-22	6.0224	USD-HKD-EUR-USD	1.031	HKD/USD USD/EUR	EUR/HKD
2008-9-23	6.0224	USD-EUR-HKD-USD	0.382	EUR/USD HKD/EUR	USD/HKD
2008-9-24	6.0225	USD-GBP-HKD-USD	0.390	GBP/USD HKD/GBP	USD/HKD
2008-9-25	6.0228	USD-GBP-HKD-USD	1.087	GBP/USD HKD/GBP	USD/HKD
2008-9-26	6.0225	USD-GBP-SGD-USD	0.351	GBP/USD SGD/GBP	USD/SGD
2008-9-29	6.0225	USD-GBP-JPY-USD	1.952	GBP/USD USD/JPY	JPY/GBP
2008-9-30	6.0233	EUR-JPY-SGD-EUR	1.412	JPY/EUR EUR/SGD	SGD/JPY
2008-10-1	6.0228	EUR-GBP-HKD-EUR	1.947	GBP/EUR HKD/GBP	EUR/HKD
2008-10-2	6.0228	EUR-GBP-SGD-EUR	0.403	GBP/EUR SGD/GBP	EUR/SGD
2008-10-3	6.0231	EUR-SGD-GBP-EUR	0.630	SGD/EUR GBP/SGD EUR/GBP	
2008-10-6	6.0231	EUR-SGD-JPY-EUR	2.824	SGD/EUR JPY/SGD EUR/JPY	
2008-10-7	6.024	USD-EUR-JPY-USD	1.026	EUR/USD JPY/EUR	USD/JPY
2008-10-8	6.024	USD-GBP-HKD-USD	1.905	GBP/USD HKD/GBP	USD/HKD
2008-10-9	6.0245	USD-EUR-JPY-USD	1.362	EUR/USD JPY/EUR	USD/JPY
2008-10-10	6.0249	USD-EUR-GBP-USD	0.786	EUR/USD GBP/EUR USD/GBP	
2008-10-13	6.0254	USD-JPY-SGD-USD	2.752	JPY/USD USD/SGD	SGD/JPY
2008-10-14	6.0237	EUR-SGD-GBP-EUR	0.738	SGD/EUR GBP/SGD	EUR/GBP
2008-10-15	6.0241	USD-EUR-JPY-USD	2.292	EUR/USD JPY/EUR	USD/JPY
2008-10-16	6.0246	USD-JPY-EUR-USD	1.768	JPY/USD USD/EUR	EUR/JPY
2008-10-17	6.024	EUR-GBP-HKD-EUR	0.465	GBP/EUR HKD/GBP	EUR/HKD
2008-10-20	6.024	EUR-GBP-HKD-EUR	1.968	GBP/EUR HKD/GBP	EUR/HKD
2008-10-21	6.0242	USD-GBP-HKD-USD	2.515	GBP/USD HKD/GBP	USD/HKD
2008-10-22	6.0249	USD-GBP-JPY-USD	2.403	GBP/USD JPY/GBP USD/JPY	
2008-10-23	6.0257	USD-JPY-EUR-USD	0.999	JPY/USD USD/EUR	EUR/JPY
2008-10-24	6.0263	EUR-GBP-JPY-EUR	3.770	GBP/EUR EUR/JPY	JPY/GBP

	λmax	Path	Profit(%)	Underpriced	Overpriced
2008-10-27	6.027	EUR-GBP-HKD-EUR	0.994	GBP/EUR HKD/GBP	EUR/HKD
2008-10-28	6.028	USD-JPY-GBP-USD	5.953	JPY/USD USD/GBP	GBP/JPY
2008-10-29	6.0258	USD-HKD-GBP-USD	1.224	HKD/USD GBP/HKD	USD/GBP
2008-10-30	6.0253	EUR-GBP-HKD-EUR	1.862	GBP/EUR HKD/GBP	EUR/HKD
2008-10-31	6.0255	USD-GBP-SGD-USD	1.181	GBP/USD SGD/GBP	USD/SGD
2008-11-3	6.0256	USD-GBP-HKD-USD	3.802	GBP/USD HKD/GBP USD/HKD	
2008-11-4	6.0255	EUR-HKD-GBP-EUR	2.398	HKD/EUR GBP/HKD EUR/GBP	
2008-11-5	6.0251	USD-EUR-JPY-USD	2.377	EUR/USD JPY/EUR USD/JPY	
2008-11-6	6.0256	USD-GBP-HKD-USD	1.616	GBP/USD HKD/GBP	USD/HKD
2008-11-7	6.0261	USD-EUR-HKD-USD	0.518	EUR/USD HKD/EUR	USD/HKD
2008-11-10	6.0254	USD-EUR-JPY-USD	1.075	EUR/USD JPY/EUR USD/JPY	
2008-11-11	6.026	USD-GBP-HKD-USD	1.442	GBP/USD HKD/GBP	USD/HKD
2008-11-12	6.0265	USD-GBP-JPY-USD	4.065	GBP/USD USD/JPY	JPY/GBP
2008-11-13	6.027	USD-JPY-EUR-USD	3.562	JPY/USD USD/EUR	EUR/JPY
2008-11-14	6.0266	USD-EUR-SGD-USD	0.778	EUR/USD SGD/EUR	USD/SGD
2008-11-17	6.0269	EUR-HKD-GBP-EUR	1.623	HKD/EUR GBP/HKD EUR/GBP	
2008-11-18	6.0268	EUR-SGD-GBP-EUR	0.497	SGD/EUR GBP/SGD	EUR/GBP
2008-11-19	6.0268	USD-EUR-JPY-USD	2.340	EUR/USD JPY/EUR	USD/JPY
2008-11-20	6.0269	USD-EUR-JPY-USD	1.394	EUR/USD JPY/EUR USD/JPY	
2008-11-21	6.0276	USD-JPY-EUR-USD	1.399	JPY/USD USD/EUR	EUR/JPY
2008-11-24	6.0279	USD-JPY-SGD-USD	3.204	JPY/USD USD/SGD	SGD/JPY
2008-11-25	6.0266	USD-HKD-GBP-USD	2.327	HKD/USD GBP/HKD USD/GBP	
2008-11-26	6.0268	EUR-SGD-GBP-EUR	0.522	SGD/EUR GBP/SGD	EUR/GBP
2008-11-27	6.0267	USD-GBP-EUR-USD	0.201	GBP/USD USD/EUR	EUR/GBP
2008-11-28	6.0267	EUR-GBP-HKD-EUR	0.479	GBP/EUR HKD/GBP	EUR/HKD

Table 3-3 Arbitrage without bid-ask spread

	λmax	Path	Profit(%)	maximum bid-ask spread	Underpriced	Overpriced
2008-9-1	6*	EUR-GBP-JPY-EUR	0.948	0.248	GBP/ EUR JPY/GBP EUR/JPY	
2008-9-2	6*	HKD-USD-GBP-HKD	1.294	0.318	USD/HKD GBP/USD HKD/GBP	
2008-9-3	6*	GBP-JPY-USD-GBP	0.952	0.249	USD/JPY GBP/USD	JPY/GBP
2008-9-4	6.0001	GBP-JPY-USD-GBP	2.687	0.544	JPY/GBP USD/JPY GBP/USD	
2008-9-5	6*	JPY-EUR-USD-JPY	1.207	0.301	EUR/JPY USD/EUR	JPY/USD
2008-9-8	6*	GBP-JPY-USD-GBP	1.849	0.417	JPY/GBP USD/JPY GBP/USD	
2008-9-9	6*	JPY-USD-SGD-JPY	1.000	0.259	USD/JPY SGD/USD JPY/SGD	
2008-9-10	6*	GBP-HKD-USD-GBP	0.874	0.233	HKD/GBP USD/HKD GBP/USD	
2008-9-11	6*	JPY-EUR-SGD-JPY	1.368	0.333	SGD/EUR JPY/SGD	EUR/JPY
2008-9-12	6.0001	GBP-USD-JPY-GBP	2.677	0.543	USD/GBP JPY/USD GBP/JPY	
2008-9-15	6.0001	JPY-USD-HKD-JPY	2.631	0.537	USD/JPY HKD/USD	JPY/HKD

	λmax	Path	Profit(%)	maximum bid-ask spread	Underpriced	Overpriced
2008-9-16	6*	JPY-HKD-USD-JPY	1.266	0.313	HKD/JPY USD/HKD JPY/USD	
2008-9-17	6.0001	JPY-USD-HKD-JPY	1.988	0.440	USD/JPY HKD/USD	JPY/HKD
2008-9-18	6*	JPY-HKD-USD-JPY	0.681	0.189	HKD/JPY USD/HKD	JPY/USD
2008-9-19	6.0001	GBP-USD-JPY-GBP	2.469	0.513	USD/GBP JPY/USD GBP/JPY	
2008-9-22	6*	JPY-USD-HKD-JPY	1.326	0.325	USD/JPY HKD/USD	JPY/HKD
2008-9-23	6*	GBP-HKD-JPY-GBP	1.049	0.270	HKD/GBP GBP/JPY	JPY/HKD
2008-9-24	6*	JPY-HKD-USD-JPY	0.680	0.189	HKD/JPY USD/HKD	JPY/USD
2008-9-25	6*	GBP-HKD-USD-GBP	1.274	0.3150	HKD/GBP USD/HKD GBP/USD	
2008-9-26	6*	HKD-GBP-EUR-HKD	1.522	0.3612	GBP/HKD HKD/EUR	EUR/GBP
2008-9-29	6.0001	GBP-JPY-USD-GBP	4.124	0.724	JPY/GBP USD/JPY GBP/USD	
2008-9-30	6.0001	EUR-JPY-SGD-EUR	2.634	0.537	JPY/EUR SGD/JPY EUR/SGD	
2008-10-1	6.0001	EUR-GBP-HKD-EUR	2.069	0.453	GBP/EUR HKD/GBP EUR/HKD	
2008-10-2	6*	EUR-SGD-JPY-EUR	1.580	0.371	SGD/EUR JPY/SGD EUR/JPY	
2008-10-3	6*	EUR-JPY-GBP-EUR	1.261	0.312	JPY/EUR GBP/JPY EUR/GBP	
2008-10-6	6.0003	EUR-GBP-JPY-EUR	5.059	0.823	GBP/EUR JPY/GBP EUR/JPY	
2008-10-7	6*	EUR-JPY-GBP-EUR	1.801	0.409	JPY/EUR GBP/JPY EUR/GBP	
2008-10-8	6.0001	GBP-JPY-USD-GBP	3.703	0.675	JPY/GBP USD/JPY GBP/USD	
2008-10-9	6*	JPY-USD-EUR-JPY	1.433	0.344	USD/JPY EUR/USD JPY/EUR	
2008-10-10	6*	JPY-HKD-USD-JPY	1.325	0.324	HKD/JPY USD/HKD	JPY/USD
2008-10-13	6.0002	GBP-USD-JPY-GBP	5.185	0.835	USD/GBP JPY/USD GBP/JPY	
2008-10-14	6*	HKD-GBP-EUR-HKD	1.655	0.384	EUR/GBP HKD/EUR	GBP/HKD
2008-10-15	6.0001	JPY-USD-GBP-JPY	3.411	0.640	USD/JPY GBP/USD JPY/GBP	
2008-10-16	6*	JPY-SGD-USD-JPY	1.921	0.429	USD/SGD JPY/USD	SGD/JPY
2008-10-17	6*	EUR-GBP-JPY-EUR	0.752	0.205	GBP/EUR EUR/JPY	JPY/GBP
2008-10-20	6*	EUR-GBP-HKD-EUR	2.084	0.455	GBP/EUR HKD/GBP EUR/HKD	
2008-10-21	6.0002	GBP-JPY-USD-GBP	4.496	0.764	JPY/GBP USD/JPY GBP/USD	
2008-10-22	6.0002	GBP-JPY-USD-GBP	4.962	0.813	JPY/GBP USD/JPY GBP/USD	
2008-10-23	6*	JPY-EUR-USD-JPY	1.072	0.274	USD/EUR JPY/USD	EUR/JBP
2008-10-24	6.0004	EUR-GBP-JPY-EUR	6.561	0.962	GBP/EUR JPY/GBP EUR/JBP	
2008-10-27	6.0001	EUR-GBP-JPY-EUR	3.520	0.653	GBP/EUR JPY/GBP EUR/JBP	
2008-10-28	6.0006	GBP-USD-JPY-GBP	8.936	1.149	USD/GBP JPY/USD GBP/JPY	
2008-10-29	6*	GBP-SGD-JPY-GBP	1.879	0.422	SGD/GBP JPY/SGD GBP/JPY	
2008-10-30	6.0001	EUR-GBP-JPY-EUR	2.546	0.524	GBP/EUR JPY/GBP EUR/JPY	
2008-10-31	6*	GBP-JPY-USD-GBP	1.351	0.329	USD/JPY GBP/USD	JPY/GBP
2008-11-3	6.0002	HKD-USD-GBP-HKD	3.927	0.701	USD/HKD GBP/USD HKD/GBP	
2008-11-4	6.0001	EUR-HKD-GBP-EUR	2.502	0.518	HKD/EUR GBP/HKD EUR/GBP	
2008-11-5	6.0001	EUR-JPY-USD-EUR	2.456	0.511	JPY/EUR USD/JPY EUR/USD	
2008-11-6	6.0001	GBP-JPY-USD-GBP	2.662	0.541	JPY/GBP USD/JPY GBP/USD	
2008-11-7	6.0001	EUR-JPY-GBP-EUR	2.478	0.515	JPY/EUR EUR/GBP	GBP/JPY
2008-11-10	6.0001	GBP-JPY-USD-GBP	2.632	0.537	JPY/GBP USD/JPY GBP/USD	
2008-11-11	6*	GBP-HKD-USD-GBP	1.508	0.358	HKD/GBP USD/HKD GBP/USD	

	λmax	Path	Profit(%)	maximum bid-ask spread	Underpriced	Overpriced
2008-11-12	6.0004	GBP-JPY-USD-GBP	6.956	0.996	JPY/GBP USD/JPY GBP/USD	
2008-11-13	6.0001	JPY-EUR-USD-JPY	3.639	0.667	EUR/JPY USD/EUR JPY/USD	
2008-11-14	6*	EUR-SGD-JPY-EUR	0.944	0.248	SGD/EUR JPY/SGD EUR/JPY	
2008-11-17	6.0001	EUR-JPY-GBP-EUR	2.071	0.453	GBP/JPY EUR/GBP	JPY/EUR
2008-11-18	6*	EUR-JPY-GBP-EUR	1.255	0.311	JBP/EUR GBP/JPY EUR/GBP	
2008-11-19	6*	EUR-JPY-USD-EUR	2.419	0.506	JBP/EUR USD/JPY EUR/USD	
2008-11-20	6.0001	GBP-JPY-USD-GBP	3.560	0.658	JBP/GBP USD/JPY GBP/USD	
2008-11-21	6.0001	GBP-USD-JPY-GBP	2.894	0.573	USD/GBP GBP/USD GBP/JPY	
2008-11-24	6.0003	JPY-GBP-USD-JPY	6.117	0.923	GBP/JPY USD/GBP JBP/USD	
2008-11-25	6.0001	GBP-USD-HKD-GBP	2.397	0.503	USD/GBP HKD/USD GBP/HKD	
2008-11-26	6*	EUR-SGD-GBP-EUR	0.598	0.169	SGD/EUR GBP/SGD EUR/GBP	
2008-11-27	6*	EUR-JPY-GBP-EUR	0.710	0.195	JBP/EUR EUR/GBP	GBP/JPY
2008-11-28	6*	EUR-GBP-JPY-EUR	0.665	0.185	GBP/EUR JBP/GBP EUR/JPY	

From matrix C, we can find which currency overpriced or underpriced. Now, we try to find an arbitrage path. we convert EUR into GBP, where GBP is underpriced, then convert GBP into HKD, where HKD is underpriced. At last we convert HKD into EUR to close the route and realize the profit. $1/0.9943^{*}1/0.996301^{*}1/1.005913-1=0.35328\%$. Even though this profit is calculated from the hypothetical matrix C, we can show that it is equal to the real profit computed using real-world exchange rate from matrix A. To see this, suppose that we followed the path EUR -> GBP -> HKD -> EUR, using the rates quoted in matrix A. The profit from this transaction (taking in the effect of bid-ask spread) is (1 / 1.233198 / 0.070947 / 11.3894) – 1 = 0.35328%, which is equal to the profit computed from the numbers in matrix C.

3.3 Result

Result presented in two tables. Table 3-2 shows the arbitrage with bid-ask spread and Table 3-3 shows the arbitrage without bid-ask spread.

4 Conclusion

This article gathers data of 65 days in three month to find the arbitrage opportunity with three different currencies. If ignoring the sequence of conversion, there are 40 path to get the profit. we pick the maximum profit from everyday to analyze the arbitrage opportunity. (there is no multi-point arbitrage if no three-point arbitrage. The less the currency, the shorter the path the shorter the path, the more profit with the bid-ask spread.). in this article, we do not consider the borrowing cost, which means we assume no bid-ask spread in the money market.

- In this article, we observe (although we still cannot prove rigorously) that the sufficient condition for the existence of arbitrage path, $\lambda max \neq n$, still holds even for the case of nonzero bid-ask spread.

- From the data of three month, we found that if the maximum bid-ask spreads are, on average, less than 0.47753% (average of 65 days), there will be opportunity for arbitrage profit.

- we also find the pair of currencies that appears most often, which can tell the investor the optimal path for the arbitrage. Under the situation of the bid-ask spread, the GBP-HKD (HKD underpriced with respect to GBP) appears 54 times in 65 days and the USD-GBP (GBP underpriced with respect to USD) appears 48 times. Without the bid-ask spread, the GBP-JPY (JPY underpriced with respect to GBP) and JPY-USD (USD underpriced with respect to JPY) appear 60 times, the USD-GBP appears 54 times. It should be noted, however, that the arbitrage opportunities that we found may be a result of asynchronous data. Nevertheless, the modified matrix method presented in this article can be used on tick data to determine an arbitrage path in real time.

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